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A modeling framework for the estimation of optimal CO_2 emission taxes for private transport

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Abstract

In this paper, a novel modeling framework is proposed for the estimation of optimal CO_2 emission taxes for urban traffic. The framework is based on a bi-level model comprising a combined equilibrium model with elastic demand and a "pollution taxes" (PTs) estimation model based on vehicle kilometers traveled and emissions produced. A bi-level optimization problem is proposed for the PT estimation model (PTM) in order to provide the minimum price which reduces emissions generated in an urban area to a desired value dependent on the environmental goals. To solve this problem, the *Regula Falsi* method is proposed and it exhibits a high enough rate of convergence. Two tests using the Nguyen and Dupuis network and Barcelona network (Spain) have been performed to test the convergence of our resolution method and the applicability of the proposal over networks with different sizes. The results are very promising and allow the implicit definition of the behavior of users against different PT prices.

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Keywords: CO_2 emissions; pricing; elastic demand marginal; abatement cost curves; private transport; combined equilibrium models;

1. Introduction

Since the implementation of *the Kyoto protocol* on December 11th, 1997, industrialized countries have had to reduce their polluting emissions in order to meet the international commitments set in the Kyoto Protocol Reference Manual (U.N.F.C.C.C.,2008). Efficient mechanisms for reducing emissions are *International emissions-trading markets* in which the rights of emissions are bought/sold. This scheme exploits differing Marginal Abatement Cost Curves (MACs) in the productive sectors to achieve the goal of reducing the emissions and the economic costs. Each firm can either reduce all the required amount of emissions by itself or it can choose to buy or sell them in the market.

As well as other industrial sectors, transportation is one of the few sectors where emissions are still growing. Nowadays almost 26% of global CO_2 emissions are produced by transportation, in which the private one represents

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one of the largest shares. Because of their importance, these emissions should participate in international markets to achieve environmental objectives proposed. There are many difficulties for this, like the problem of its implementation and the user acceptance. Users of private vehicles are micro-producers of emissions while the markets are designed primarily for large companies (emissions macro-producers). One way to implement this scheme would be that certain public entities (PE) were responsible for the emissions in certain areas, encouraging their participation. The PEs should participate in emission's market directly and should be involved in the impact of "pollution taxes"' (PTs) when using private transport (either for commercial or personal use) to reduce/finance the acquisition of rights of emissions. In this process a key element is that each PE should know the MAC of its area in order to properly manage the purchase/sale of emission rights.

In this paper, we propose an analytical tool to obtain MACs based on a mathematical program with equilibrium constraints with different types of users. The goal of this work is to give an answer in this context to the question: "How much PT must be charged to a single user in order to meet the environmental goals for a particular city?".

Literature studies have proposed several approaches for road pricing problem (Schaller, 2010), different from the presented in this paper. These approaches of tolling try to reduce the traffic congestion and negative effects, as the air pollution, in certain urban areas. One example is the London congestion charging scheme implemented in February 2003 (Beevers, Carslaw, 2005). The main idea of this paper is to take into account the environmental costs in the private transport costs in order to connect environmental objectives with consumer objectives.

Dealing with several questions arise:

- How are PTs accepted by the users and how do they affect their behavior?

Several researchers have investigated traffic assignment models using environmental cost functions. An overview of this topic is out of the scope of this work and a brief review can be found in publication of Ahn and Rakha (2008). These models combine an assignment model with a simplified emissions model. The main objective is to analyze routing strategies to reduce emissions with a fixed demand. In this paper the key aspect to be considered in the traffic model is the reaction of users to the new PT. The relationship considered by users between the number of journeys and the taxes applied is not linear. Assuming normal action during one week, users make various journey plans using private transport: taking children to school, going to work, going to visit relatives, going to the countryside, etc. When PTs are higher, some users will not accept the cost and will reduce the number of journeys (e.g. fewer visits to relatives) or will change from private to public alternatives (e.g. going to work is mandatory but some users can use public transport).

Considering these situations, we consider *combined equilibrium models* (Oppenheim, 1995; Zhou, Chen, Wong 2009) as a powerful tool to answer this question. These models simulate the behavior of the users and can represent the elasticity of the demand with respect to new taxes. When a single user makes a trip using her private car, the total quantity of emissions can be estimated by using previously collected data generated by the equilibrium model. Considering users' behavior, when users deal with a new PT their reaction depends on the user type. The perceived travel cost is different when different user types are considered, e.g. a higher-earning person can pay higher taxes than a lower earner. This, in addition to the need to estimate emissions depending on vehicle features, has led us to consider the general multi-user equilibrium model proposed in paper of Cantarella (1997).

- How much PT must a single user pay depending on emissions?

The exact measurement of how much PT must be charged depending on a specific quantity of emissions. There is no generally accepted method (Smit, Ntziachristos, Boulter, 2010). The so-called *average speed* models like COPERT (Ekström, Sjödin, Andreasson, 2004), MOBILE (National Research Council, 2000), are widely applied in practice. In these models, the emission factors (g km^{-1}) are stated as a function of average speed. Emission from traffic are calculated by multiplication of emission factors with the vehicles kilometres travelled (KVT) for different vehicles classes.

The mechanisms on which our proposal is based are the following:

1. Every kilogram of emissions must be paid at the same price. This price must be independent of the vehicle which has generated it (e.g. an ecological car against an SUV).
2. The price of a single kilo of emissions must be fixed by the environmental goals which are sought. The price will be the lowest tax price which will secure the meeting of the environmental policy.

- Which are the most suitable PT mechanisms to charge?

Nowadays, it is very difficult to implement "in situ" tax policies. This is a common problem in pricing estimation, and it has led to the emergence of the so-called first and second tolls (Yildirim, Hearn, 2005; Gardner, Unnikrishnan, Waller, 2009). Because of this, the usual mechanism on emission pricing is to charge depending on the vehicle kilometers travelled and/or the vehicle characteristics.

For our purposes, the estimation of PTs is made in terms of the emissions produced. These emissions are calculated with a generic traffic emission model based on the output of a combined equilibrium model and are employed for estimating the optimal PT. Finally the mechanism used to charge this PT to the users has been developed depending on the technical features of the vehicles and the kilometers travelled by them.

This paper is structured as follows: Section 2 describes the modeling framework employed and defines the problem of PT estimation in terms of a mathematical program with equilibrium constraints, Section 3 proposes a solution method, and Section 4 shows the computational tests performed to test the applicability of our proposal with the real Barcelona network and a small network (Nguyen, Dupuis, 1984). Finally, Section 5 summarizes the main contributions of this paper and proposes new improvements.

2. A model for the estimation of optimal CO2 emission taxes

Figure 1 shows the structure of analytical tools used to calculate PT.

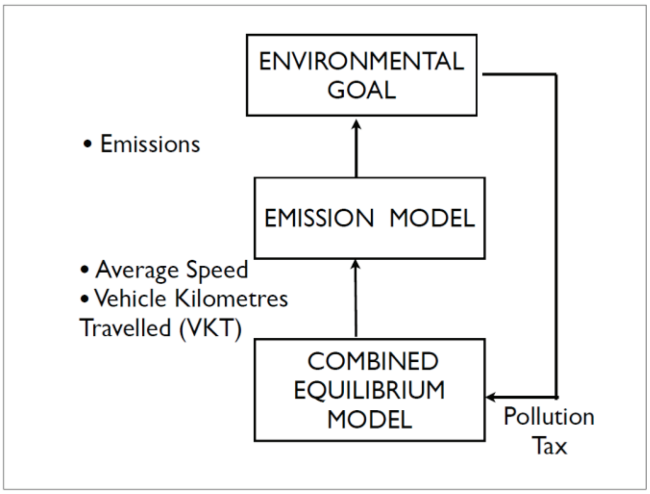

Figure 1 Scheme of PT model

In this paper we consider the general approach to multi-user equilibrium assignment with elastic demand (combined equilibrium model) given by Cantarella (1997). This model is formulated as a fixed-point problem in the link flow space, over the set of feasible link flow patterns. In this section we briefly review this approach and we show that the model allows us to introduce systematically the emissions tax in the route utility. These taxes affect the level of demand and the choice of route in the network.

Users are assumed to be grouped into classes, such that all the users of a *class* share all behavioral characteristics and their vehicle has similar characteristics with respect to amount of emissions they produce.

The following table shows, for convenience, the variables used in the formulation of a bi-level model for PT estimation (PTM).

Table 1. Notation used to formulate PT model

n be the number of links of the network.
f be the (nx1) link flow vector.
K_i be the set of relevant routes for class i.
$m_i=\lvert K_i \rvert$ be the number of relevant routes for class i.
$\mathbf{B_i}$ be the (n x m_i) matrix mapping all relevant routes for class i to all links of the networks.
d_i be the level of demand for class i.
$\mathbf{g_i}$ be the (m_i x1) route *link-wise additive cost* vector for class i.
$\mathbf{r_i}$ be the (m_i x1) route *emission cost* vector for class i.
$\mathbf{v_i}$ be the (m_i x1) route systematic utility vector for class i.
$\mathbf{p_i}$ be the (m_i x1) route choice probability vector for class i.
$\mathbf{h_i}$ be the (m_i x1) route flow vector for class i.
P be the price applied to the emission of a single unit of pollutant.
P_i be the price applied to the emission per kilometer travelled to users from type i.
$\mathbf{L_i}$ be the length (kilometers) route vector for class i.
β_i, α_i be the parameters which allow the homogenization between prices and perceived travel costs.
$E_i(\mathbf{h}^*_i, \boldsymbol{c}^*)$ quantity of emissions generated in the network for class i and associated with an equilibrium vector $\mathbf{h}^*_i$ and depending on the link travel cost $\mathbf{c}^*$.
E_0 be the maximum allowed emissions to meet environmental goals.

Let d_i be the space of feasible demands of users of type i to be seen in the traffic network. Any non-negative vector $\mathbf{h_i}$ is a *feasible* route flow vector, if demand conservation is satisfied, $\mathbf{1}^T\mathbf{h}= d_i$. Let

$$H_i = \{\mathbf{h}_i \geq \mathbf{0}, \mathbf{1}^T \mathbf{h} = d_i \in D_i\} \subset \Re^{mi} \quad (1)$$

be the set of feasible route flows. The consistency between route and link flows yields

$$\mathbf{f} = \sum_i \mathbf{B}_i \mathbf{h}_i \quad (2)$$

where $\boldsymbol{B}_\mathbf{i}\, \mathbf{h_i}$ are the link flows for class i. A link flow vector $\boldsymbol{f}$ is *feasible* if it is feasible for all classes through (2).

Let

$$F = \left\{ \mathbf{f} = \sum_i \mathbf{B}_i \mathbf{h}_i, \mathbf{h}_i \in H_i, \forall i \right\} \subset \Re^n \quad (3)$$

be the set of feasible link flows.

The effect of congestion is modeled by the *link cost-flow* function

$$\mathbf{c} = \boldsymbol{c}(\mathrm{f}) \quad (4)$$

The link and link-wise additive route cost consistency yields

$$\mathbf{g}_i = \mathbf{B}_i^T \mathbf{c} \quad \forall i \quad (5)$$

The following route *linearly systematic utility* is adopted

$$\mathbf{v}_i = -\beta_i \mathbf{g}_i - \alpha_i \mathbf{r}_i \quad \forall i \quad (6)$$

where β_i, α_i are two parameters depending on the class i which homogenize the travel cost and emission cost. The tariff model is based on two key factors:

1. That each kilo of emissions should be paid at the same rate regardless of the type of user that generates it.
2. That the tax be applied to the number of kilometers travelled; this assumption leads us to the emission route which can be expressed as:

$$\mathbf{r}_\mathrm{i} = \mathrm{P}_\mathrm{i}\, \mathbf{L}_\mathbf{i} \quad \forall i \tag{7}$$

where P_i is the emission tax per kilometer for users of class i and $\mathbf{L_i}$ is the route length vector for class i. The consistency between tax per kilometer for class i and the emission tax yields

$$P_i = \frac{E_i(\mathbf{c},\mathbf{h}_i)}{\mathbf{h}_i^T \mathbf{L}_i} P \tag{8}$$

where $E_i(\boldsymbol{c},\boldsymbol{h}_i)$ is the total emissions generated by class i which depends on type of vehicle of class i, the kilometers travelled, average speed, level of demand, etc. The analytical expression E_i depends on the emission model chosen, possible options could be MOBILE, COPERT etc.

Relations (5)-(8) may be written synthetically as

$$\mathbf{v}_\mathrm{i} = v_i\,(\mathbf{c}, P) \tag{9}$$

The perceived utility can be specified as the sum of the above *systematic* utility and a *random residual* x_i which models factors like dispersion among users and user perception errors. The so-called *satisfaction* is defined as:

$$s_i = E_\xi\left[\max(v_i + \xi_i)\right] \ \forall i \tag{10}$$

and it depends on the systematic utilities

$$\mathrm{s}_\mathrm{i} = s_i\,(\mathbf{v_i}) \tag{11}$$

The level of demand depends on route choice satisfaction, as expressed by *demand function*

$$\mathrm{d}_\mathrm{i} = d_i\,(s_i) = d_i\,(s_i\,(\mathbf{v_i})) \tag{12}$$

The probability p_k that a decision maker of the class i choose the route k within set K_i is given by the probability that the *perceived utility* u_k of this route is greater than the perceived utility of any other alternative:

$$p_k = P_r\left[v_k + \xi_k \geq v_j + \xi_j \quad j \in K_i\right] \tag{13}$$

and the choice probabilities depend on the values of systematic utility through the *route choice map*, whose explicit expression depends on the random residual joint distribution as:

$$\mathbf{p}_\mathrm{i} = p_i\,(\mathbf{v_i}) \quad \forall\ \mathbf{v_i} \in \Re^{m_i} \tag{14}$$

For each class i, the consistency among route flows, route choice probabilities and level of demand yields

$$\mathbf{h_i} = d_i\, \mathbf{p_i} \tag{15}$$

The relation between link flows and costs, called the *network loading map* (NLM), is obtained by combining the above equations

$$\mathbf{f} = f(\mathbf{c}, P) = \sum_i d_i(s_i(v_i(\mathbf{c}, P)))\mathbf{B}_i p_i(v_i(\mathbf{c}, P)) \tag{16}$$

The multi-user equilibrium assignment with elastic demand can be stated through NLM and the link cost-flow function

$$\mathbf{c}^* = c(\mathbf{f}^*) \tag{17}$$

$$\mathbf{f}^* = f(\mathbf{c}^*, P) \tag{18}$$

and the multi-user equilibrium assignment with elastic demand can be expressed as a fixed-point problem in the link flow space

$$\mathbf{f}^* = f(c(\mathbf{f}^*), P) \quad \mathbf{f}^* \in F \subset \Re^n \tag{19}$$

where $f(\cdot)$ is the network loading map and $c(\cdot)$ is the link-cost functions and P is the emission tax.

The output of the traffic equilibrium model provides enough data to estimate the total quantity of emissions generated E for an emission tax P and a traffic pattern $\mathbf{f}^*$ such as

$$E(\mathbf{f}^*, P) = \sum_i E_i(\mathbf{h}_i^*, P) \tag{20}$$

where $h_i^* = d_i(s_i(v_i(c(f^*), P)))p_i(v_i(c(f^*), P))$

This emission price P will be introduced into the traffic equilibrium model in which, because of the elasticity of the demand, the traffic equilibrium pattern $\mathbf{f}^*$ will change due to the new perceived utility affected by the new PT price P. With this situation, the problem consists in meeting some environmental goals fixed by an emissions reduction. The objective is to obtain the minimum emission price P which reduces emissions to maximum amount E_0. For this goal the following optimization model is stated:

$$\begin{aligned} &\text{minimize} \quad && P, \\ &\text{subject to:} \quad && \mathbf{f}^* = f(c(\mathbf{f}^*), P) \ \mathbf{f}^* \in F \subset \Re^n \\ & && E(f^*, P) \le E_0 \end{aligned} \tag{PTM}$$

3. A numerical algorithm for solving PTP

In this section, we propose a numerical method for solving the bi-level problem PTM. PT Model implicitly defines a function $\tilde{E}(P)$ which provides the quantity of emissions as a function of the price P selected. This function represents the inverse of MAC curve. The proposed method is based on the assumption that the function $\tilde{E}(P)$ is decreasing and, therefore, the solution P^* of the bi-level model PTM will satisfy $\tilde{E}(P^*) = E_0$. With this assumption, the problem of solving the bi-level model is similar to the root-finding problem of the following function:

$$\hat{E}(P) = \tilde{E}(P^*) - E_0 \tag{21}$$

where P is the variable which represents the tax charged to the users, $\tilde{E}(P)$ is the total quantity of emissions, which depends on the variable P, and E_0 is the maximum quantity of emissions allowed when a reduction policy is applied.

The essential issue of the methods to find a root of Eq. (21) is that the function is implicitly defined and a great computational effort is needed for its evaluation. This fact is the key reason for requiring the application of solution algorithms with convergence speeds as high as possible. The proposed method is the *Regula Falsi* method described in Table 2. In the book (Householder, 1970), we can find sufficient conditions for convergence of the previously proposed methods when the two initial points are near to the root. The *Regula Falsi* method has the same structure as the classic bisection method but, instead of taking a midpoint, the new point is calculated using the secant method:

$$\begin{gathered} P_0 \text{ arbitrary} \\ P_{k+1} = P_k - \frac{P_k - P_{k-1}}{\hat{E}(P_k) - \hat{E}(P_{k-1})} \hat{E}(P_k) \end{gathered} \tag{22}$$

Table 2 shows how our proposed resolution method works.

Table 2 Regula Falsi Algorithm

0. (*Initialization*). Let P_0 and P_1 be two initial prices satisfying $\tilde{E}(P_0)\tilde{E}(P_1) < 0$ and let $\varepsilon > 0$ a tolerance parameter. Initialize $k=1$.
1. Compute
$$P_{k+1} = P_k - \frac{P_k - P_{k-1}}{\hat{E}(P_k) - \hat{E}(P_{k-1})} \hat{E}(P_k)$$
2. (*Stopping criterion*). *If* $| P_{k+1} - P_k | < \varepsilon$ then $P^* \approx \frac{P_{k+1} + P_k}{2}$ and Stop, otherwise continue with Step 3.
3. (*Combined equilibrium model*). Let P_{k+1} be PT and compute $\tilde{E}(P_{k+1})$
4. (*New pair of points*).
$$\begin{gathered} \tilde{E}(P_{k-1})\tilde{E}(P_{k+1}) < 0 \Rightarrow P_k = P_{k-1} \\ \tilde{E}(P_{k-1})\tilde{E}(P_{k+1}) > 0 \Rightarrow P_k = P_k \\ \tilde{E}(P_{k+1}) = 0 \Rightarrow P^* = P_c (Stop) \end{gathered}$$

Go to Step 1.

4. Computational experiments

In this section we perform some tests with the PTM proposed in this paper using a small network of Nguyen-Dupuis (NDN) (Nguyen &Dupuis, 1984) and Barcelona Network (BN). The NDN has 13 nodes, 19 links, and 4 OD pairs W={1, 2, 3, 4}: pair (1-2) with a demand d_0^1, (1-3) with a demand d_0^2, (4-2) with a demand d_0^3, and (4-3) with a demand d_0^4. The BN has 1,020 nodes, 2,522 links and 7,922 OD pairs.

The characteristics of these tests are: i) we have considered a single user type (i=1), ii) a deterministic choice model, (**u** = **v**) and iii) the demand function is given by the following logit model:

$$d^j = d(s^j) = \frac{\exp(-\beta s^j)}{\exp(-\beta s^j) + \exp(-\beta s_0^j)} (2 d_0^j) \tag{23}$$

where s_0^j and d_0^j are respectively the equilibrium cost and the OD demand for the pair j in the original traffic assignment model with fixed demands. This parameter selection means that the elastic equilibrium model and the inelastic equilibrium for every β are coincident for P=0. The β values represent the users' sensitivity to the applied emissions tax.

The emission cost of the route k, has been calculated

$$\alpha r_k = P\sum_{a\in k} t_a^0 \tag{24}$$

where a is a link, and t_a^0 is the travel time in the free-flow link. The pollution tax P is measured as the percentage increase in the perceived travel time in the uncongested network.

The first test was performed using network of Nguyen-Dupuis (NDN) and second test was performed using the real Barcelona network (BN). From the Barcelona data, elasticity is introduced to the model by the logit model (23).

Figure 2 shows the function $\hat{E}(P)$ for NDN when the price P of the PT is increased, for values β =*0.1* and β=*0.3*. Because the network is small we have been able to draw numerically the function $\hat{E}(P)$ to show that it is a decreasing, continuous function in this case where the *Regula Falsi* method guarantees convergence.

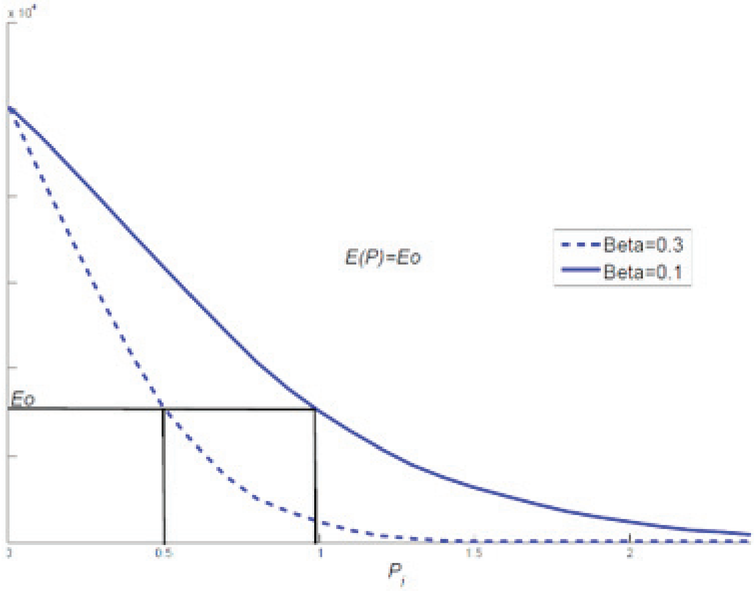

Figure 2 NDN emissions versus PT price with different β values

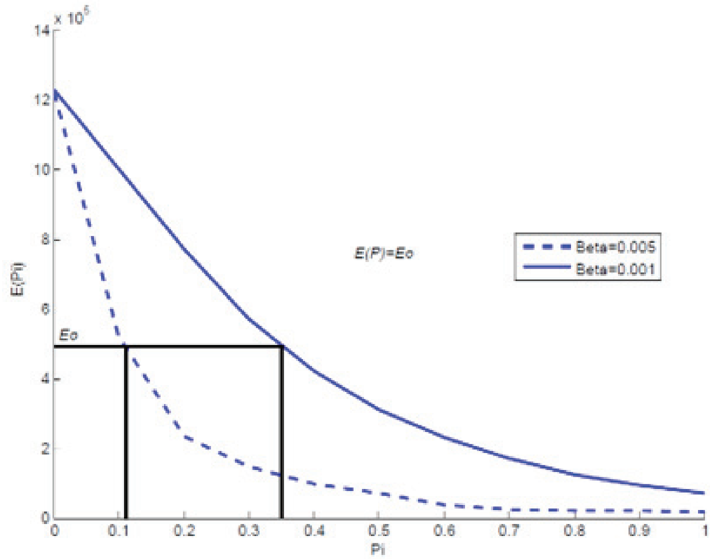

Figure 3 BN emissions versus PT price with different β values

Table 3 shows the results obtained for BN with the R*egula Falsi* method, for values β = 0.001 and β = 0.005. An approximation of the $\hat{E}(P)$ function for both values of β can be seen of Figure 3. This test has been used to test the convergence and in both cases the emissions percentage is chosen as the environmental goal of reducing emissions by 15%. It can be seen that when the value of P converges to P^* the cost of solving the equilibrium problem are greatly reduced. The equilibrium model has been solved using DSD of Larsson and Patriksson (1992), the main conclusion is that although amount of solved equilibrium problems are four and seven, the computational cost of solving the second and following problems represents approximately 30 % of the whole computational cost.

Table 3 BNT results with the *Regula Falsi* method

Iteration	P Value	Emissions	Accumulated CPU time (sec)
β =0.001			
1	0.159525	$8.63481*10^5$	13,107
2	0.080854	$1.04716*10^6$	14,975
3	0.082698	$1.04294*10^6$	16,815
4	0.082736	$1.042864*10^6$	18,672
Iteration	P Value	Emissions	Accumulated CPU time (sec)
β =0.005			
1	0.152322	$3.252627*10^5$	19,681
2	0.031136	$9.590334*10^5$	21,014
3	0.021705	$1.032779*10^6$	22,361
4	0.021029	$1.038199*10^6$	23,726
5	0.020974	$1.038633*10^6$	25,082
6	0.020970	$1.038671*10^6$	26,446
7	0.020969	$1.038672*10^6$	27,812

5. Conclusions and future work

In this paper we propose a novel approach for estimating the optimal PTs for traffic network users in order to meet environmental goals. This paper provides four fundamental contributions:

- A bi-level PTM is proposed for the estimation of the price of a single emission unit. This model shows that the price depends on the environmental goals fixed by any reduction policy. The lower level of PTM is defined with a general equilibrium assignment model and the upper level is a PT estimation model. This approach considers that any equilibrium model can be applied with the single condition of providing data about the demand equilibrium pattern such as kilometers travelled for each user type, equilibrium travel times, and average speeds. These parameters are needed to estimate the quantity of emissions depending on the technical features of the vehicles (user types). This model implicitly defines the MAC curves of private traffic emissions in a region.
- A practical implementation of the PT based on the kilometers traveled by users and the vehicle employed is proposed. The PTM would allow the estimation of the tax for each user type so that, approximately, every user pays the same price per emitted unit P^* obtained with the PTM. In order to estimate the optimal PT, we generate an estimation of the quantity of emissions produced depending on the demand equilibrium pattern.
- An algorithm based on the *Regula Falsi* method has been proposed for finding the optimal PT. The algorithm has been tested and has a fast enough convergence for real cases.
- Numerical tests with Nguyen-Dupuis and Barcelona networks have been performed. To simulate properly the behavior of the demand, elasticity has been introduced in the equilibrium model in terms of a logit-based model. With these tests we have illustrated the implicit relationship between the emissions and the PT(MACs). These tests evaluate the reluctance of the users to pay higher taxes.
- Future research will include the modeling of emission markets among regions using the MAC functions defined in this paper.

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